Paper n.1363

THE TIBURIO OF THE CATHEDRAL OF MILAN: STRUCTURAL ANALYSIS OF THE CONSTRUCTION AND 20TH CENTURY FOUNDATION SETTLEMENTS.

D. Coronelli, C. di Prisco, F. Pisanò
Politecnico di Milano
DICA, Dip. Ing. Civile Ambientale
P.za Leonardo Da Vinci 32
Milano, ITALY

S. Imposimato
FEAT, the Netherlands

S. Ghezzi, M. Pesconi
BSc Graduates
Politecnico di Milano

Abstract:
The history of the construction of the tiburio of the Cathedral of Milano is studied in relation to the load-bearing system and the foundation soil of the whole building. The structure is analysed by preliminary limit analysis. The characteristics of the tiburio and supporting piers are shown. The load path is studied, considering the construction of the supporting arches in the 15th century and the final configuration of the 18th century. An analysis of the foundation settlements caused by lowering of the water table in the 20th century is carried out by 3d finite element analysis, and compared to results from the monitoring started since the 1950s. An elastic model of the soil under the whole cathedral and the nearby buildings is used first, followed by nonlinear analyses of the soil beneath one of the main piers. The results provide an interpretation of the damage caused by the differential settlements at the end of the 20th century. The models are a basis for the prediction of future events in the structural history of the Duomo di Milano.

Keywords:
Masonry, gothic cathedrals, limit analysis, foundation settlements, finite element analysis.

INTRODUCTION

The Cathedral of Milan is one of the most important historical churches in the world, in relation to its religious, artistic and structural value. The history of the construction and 20th century restorations of the Cathedral of Milan is widely described in the works by Carlo Ferrari da Passano (e.g. Ferrari da Passano, 1988), former Architect of the Veneranda fabbrica del Duomo. The history of the structure has been analysed recently by Coronelli et al. (2014).

One of the most important features of the design is the tiburio, built above the crossing, supporting the main spire with the statue of the “Madonnina”. This part of the construction is supported on a system of arches springing from four main piers, each bearing a load of approximately 3500 tons. These members underwent a complex series of events that will be summarised in the following. Major restoration works were carried out at the end of the 20th century as a consequence of structural damage caused by soil subsidence related to lowering of the water table (wells).

The effects of the 20th century soil settlements were such that the load bearing capacity of the structure was nearly overcome. A commission of the Veneranda Fabbrica del Duomo and the Politecnico di Milano took care of the temporary propping, the explanation of the events and the repair and strengthening. The analysis of these events and the study of their solution were carried out experimentally (Ferrari da Passano and Oberti, 1983).

The study of the past events aims to a more complete understanding of the structural history and the prediction of possible future critical events. The present and future evolution is followed by monitoring. The development of analytical models gives the possibility both of understanding the past and predicting future scenarios.

For the structural behaviour the use of limit analysis provides a synthetic description of the main aspects of the response of the load bearing system (Heyman, 1995; Huerta 2001). A first global analysis of the Cathedral has been carried out by Coronelli et al. (2014); Vasic and Coronelli (2014) studied the vaults. The main events in the history of the structure of the tiburio are analysed in the present paper.

The analysis of soil settlements requires the geotechnical characterization of the soil layers under the foundation. The inhomogeneous space distribution of soil properties seems to be particularly relevant, as well as the occurrence of subsidence phenomena induced by continuous variations in the phreatic level. The apparent complexity of the problems requires to resort to numerical analyses, whose results are to be critically interpreted in the light of the data from in situ monitoring.

A first step towards the numerical analysis of soil settlements and subsidence under the Cathedral is hereafter reported, based on extensive preliminary geotechnical/geological surveys. The results from elastic analyses are then presented, by putting in evidence the effect of different assumptions concerning the (uncertain) soil compliance and the time variations of the phreatic level.

The most relevant aspect concerns the prediction of differential settlements, which are extremely important to the safety conditions of the overall structure. This will enable to forecast the occurrence in the future of dangerous critical conditions for the Cathedral.

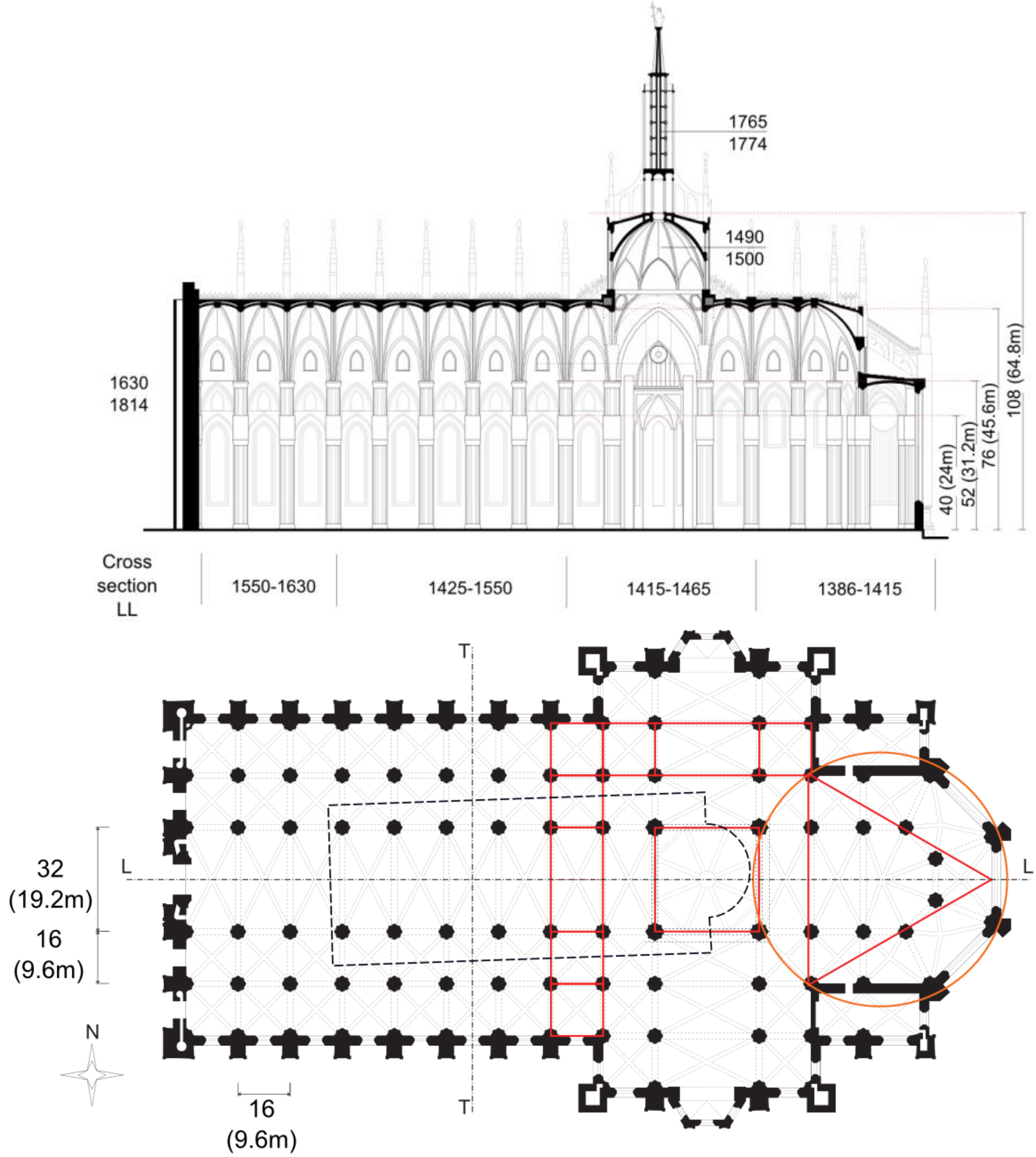


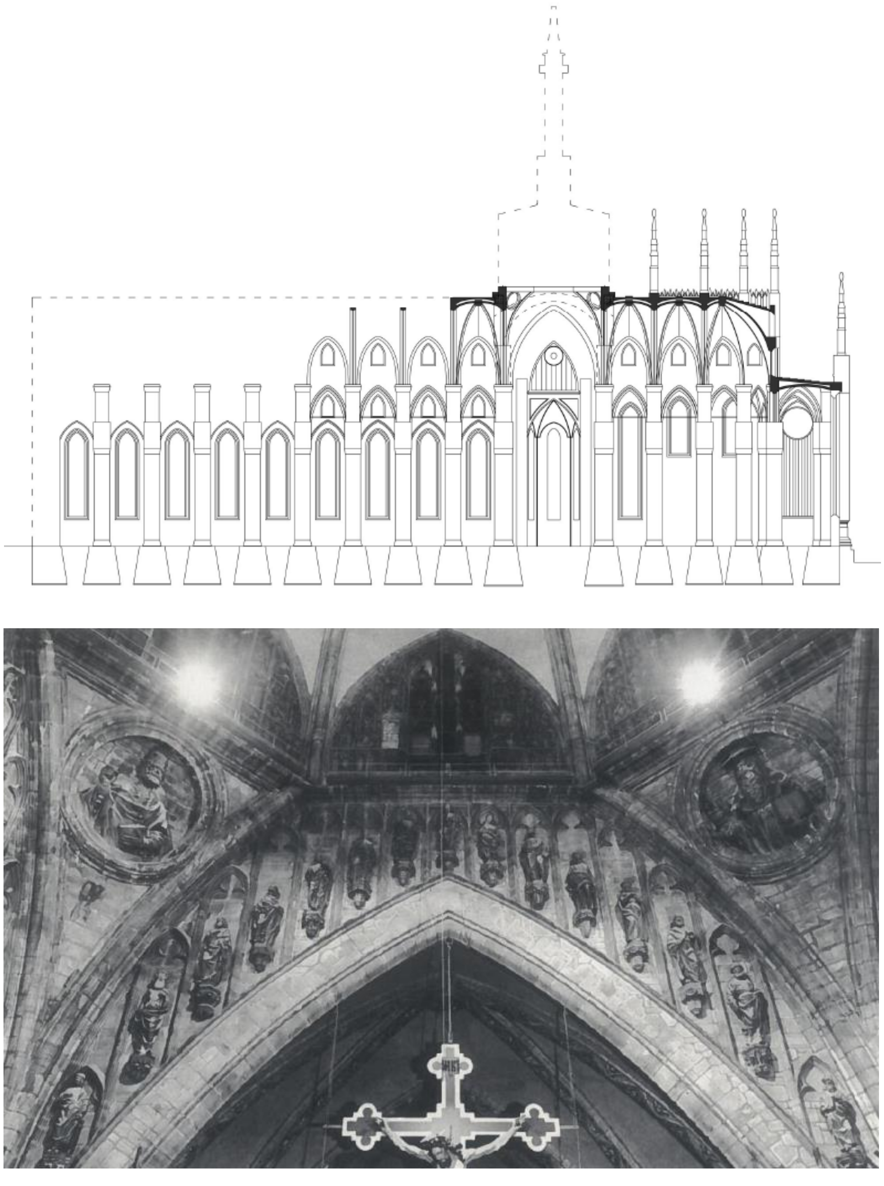
Fig.1 Geometry and history of the construction (Coronelli et al., 2014).

STRUCTURAL HISTORY

The construction of the Cathedral began in 1386. The historical phases are summarized in Fig.1, with the layout of the plan is shown in Fig.1. The size of the building places it amongst the largest churches ever built. The main nave and transept span determines a 19,2m square crossing. The highest loads are active here, due to the weight of the tiburio and main spire above. Static computations show that these members bear a load that is more than the double of those of the main nave (3500 ton compared to 1500 ton).

The cross-section of all the piers initially had to be the same (2,55m diameter). The advice of Matteo da Campione was accepted, and the four piers of the tiburio were built with a larger cross section (2,95m diameter, equivalent to a 34% increase of the cross-section).

The foundations lie 7m below the ground level. The internal piers are supported on spread foundations made of brick, whereas stone walls are along the perimeter.



(a) (b)

Fig.2 – (a) Schematic representation of the state of the construction at the moment the Tiburio was built; (b) the base of the Tiburio.

The construction proceeded from East to West with the apse and choir, and the structures designed to support the Tiburio were in place by 1450 (Fig.2a). These included the piers and vaults of the bays surrounding the crossing.

For all arches in the construction iron ties were provided. The south and north transept fronts with the stair shafts in the corners were not finished until the first part of the following century.

The project of the tiburio was discussed amongst relevant personalities such as Bramante, Da Vinci, Filarete and Nexemberg from Graz. The definitive solution was developed by Giovanni Solari (1452) followed by Guiniforte Solari (1459). The four main piers already supported pointed arches (Fig.2b), of the same shape, span (19,2m) and rise of all main arches in the nave and choir. These structures were judged insufficient and inadequate to bear the loads of the octogonal tiburio, consisting of eight support loads transferred by the dome onto a limited support area. Each arch had to bear two of these loads; the line of thrust calculated with the final loads of the construction (Coronelli et al. 2014) is definitely different from the shape of the pointed arches, and would have been external to their geometry causing the collapse of these arches (Fig.4b).

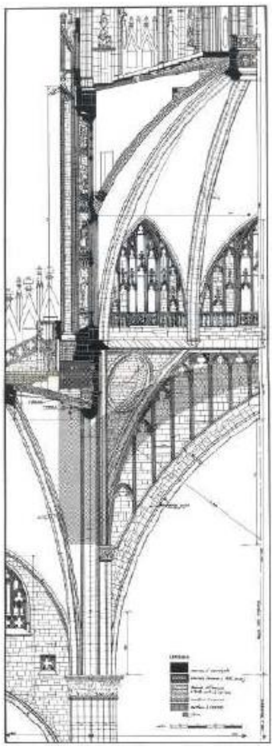
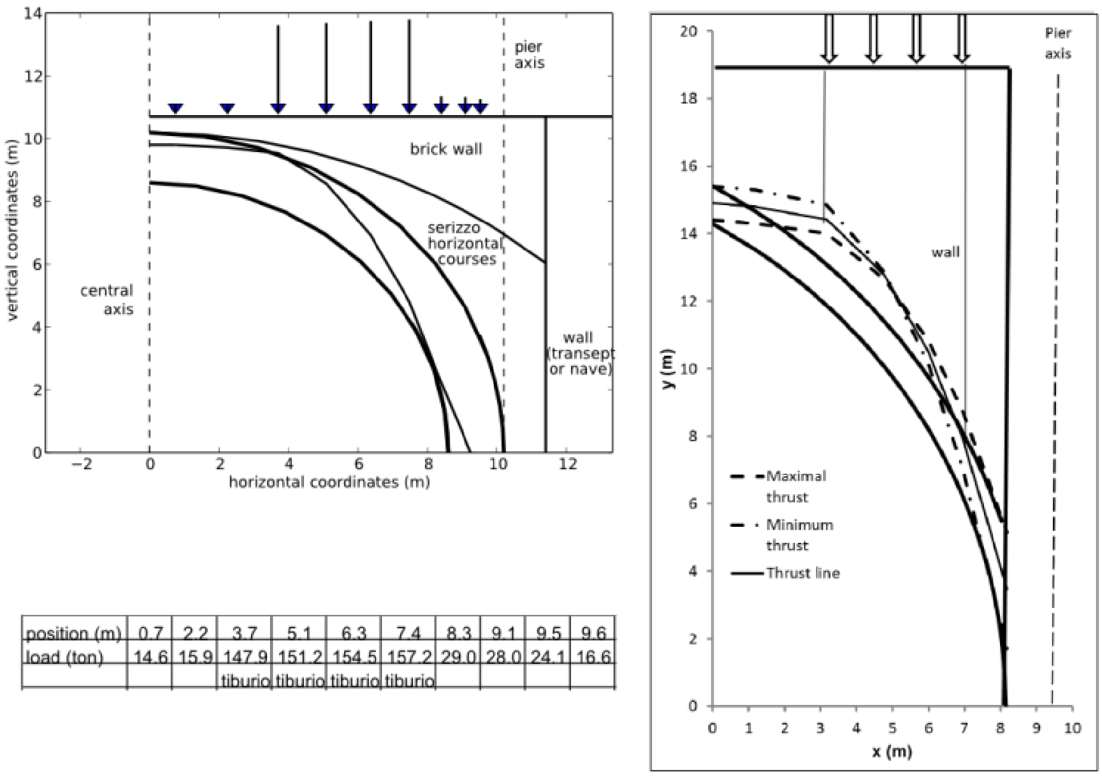


Fig.3 – Cross section of the Tiburio and supporting structures (left), with the semi-circular arches placed by Guiniforte Solari (from Ferrari da Passano and Brivio, 1967); the Tiburio, view from the East (right).

The choice made by Guiniforte Solari was to place new semicircular stone arches supported on the same four piers of the pointed arches (Fig.3a, grey in transparence). These members are stilted and set back behind the Candoglia wall above the pointed arches. Hence they are above and more external than the former structures.

One advantage of this solution was that the previous work had not to be dismantled. The walls above the gothic arches and the pendentives had already been decorated by fine sculptures (Fig.2b). Moreover the shape of the vaults in the cathedral is determined by the pointed arches (Vasic and Coronelli, 2014) hence the harmony of the geometry would have been broken by the replacement of these members.

More appealing features can be highlighted from the structural point of view. The shape is defined by an inner circular radius of 8,6m and the cross section of the arch at the key is 1,6m deep and 1,8m wide. The depth was limited by the rise of the pointed arches beneath and the position of the supports of the tiburio above (Fig.3). Above the semi-circular ring more stone courses were placed, reaching acircumference with an outer radius of 18m and centered at the springing line of the pointed arches, creating the shape shown in Fig.3, and providing a stabilising weight above the arches. The line of thrust calculated both for the self weight of the arch and with the addition of the final loads of the tiburio (Coronelli et al., 2014) is internal to the arch geometry, indicating a stable configuration (Fig.4).



position (m)	0.7	2.2	3.7	5.1	6.3	7.4	8.3	9.1	9.5	9.6
load (ton)	14.6	15.9	147.9	151.2	154.5	157.2	29.0	28.0	24.1	16.6
			tiburio	tiburio	tiburio	tiburio				

Fig.4 Thrust lines for the semi-circular (present condition, minimum thrust) and pointed arches (assumed collapse condition), with the full loads of the Tiburio (Coronelli et al., 2014).

The striking of the centerings led to major events (Ferrari da Passano and Brivio, 1967). With the thrust of the arches the supports moved outwards by 10 cm along the diagonals of the crossing; the out-of-plumb of the piers can still be measured nowadays. The ties that were in position at the haunches of the pointed arches were broken. The vaults surrounding the crossing were damaged with cracks that could still be observed in the 20th century. There is no record of damage in the piers, but Ferrari da Passano and Brivio (1967) suggest that also these members suffered. The concerns raised by these events was such that the construction was stopped around 1470.

The spread of the supports on decentering is a well-known phenomenon (Heyman, 1995). The settlement state of the arch depends on the deformability of the surrounding abutments, that have to provide the reactions balancing the thrust of the arches. In the case at study the change of configuration was such the damage described above took place. The semi-circular arches themselves di not undergo damage, and still appear perfectly sound nowadays (Ferrari da Passano and Brivio, 1967). It is should be noted that the loads were only those due to the weight of the new arches and the walls above, i.e. the Tiburio was not yet built; the thrust line without the very high and rather concentrated loads of the supports of the tiburio is internal to the semicircular arch (Coronelli et al., 2014).

The structures surrounding the tiburio were the vaults and arches described earlier in this section, supported on their piers. It can be assumed that due to the displacement of the top of the piers the rotations at the base developed eccentric reactions contributing to the new equilibrium, with the horizontal components of the thrust within these piers.

Ferrari da Passano (1988) underlined a peculiar characteristic of the semi-circular arches of the tiburio, their eccentricity in the vertical plane, and indicated this as the cause of the phenomena described above: the reactions of the arches were eccentric in the vertical plane with respect to the piers below. The spreading of the supports is the origin of the so called “minimum-thrust” condition in the arch (Heyman, 1995). When the corresponding thrust line is calculated (Coronelli et al., 2014), it is evident that the resultant reaction is quite internal within the arch cross-section at the supports (Fig.4a). Two arches are supported on each column; the sum of the two reactions is is at the middle of the line connecting the support points (Fig.6); thus the resultant reaction is not eccentric in the vertical plane, and nearly central with respect to the pier cross-section. It must be concluded thus that the damage was caused by the horizontal thrust.

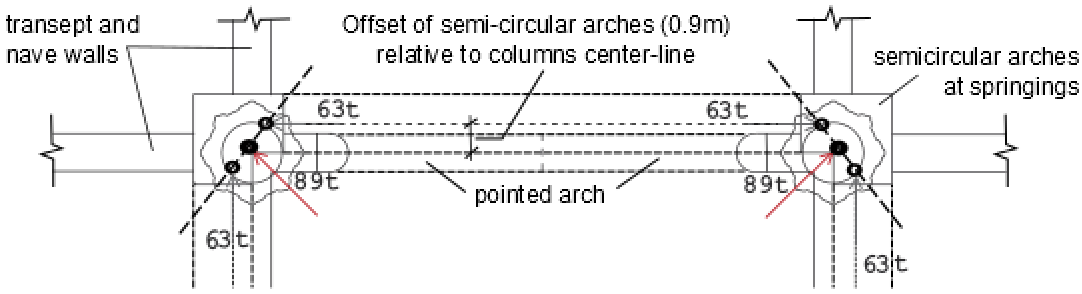


Fig.5 Position of the reactions on the piers of the Tiburio and horizontal thrust due to the loads of the semicircular arches without the Tiburio.

Guiniforte Solari was replaced in the Fabbrica del Duomo by his son-in-law and former pupil Giovanni Amadeo together with Giacomo Dolcebuono. These followers stated that the construction was stable, and the tiburio could be placed bearing on top of the structures erected by Guiniforte (Ferrari da Passano and Brivio, 1967). The construction was restarted in 1490 and finished on May 24th, 1500. As a consequence the loads added on the whole system provided an increase beneath the foundations from 1300 ton to 3200 ton.

Fig4a shows the calculation of the line of thrust in the semicircular arches supporting the weight of the whole tiburio, with a lateral thrust increasing from 89 ton (Fig.5) to 520 tons. Fig.6 shows the calculation of the line of thrust within the main pillars beneath, assuming either that (a) the thrust was not balanced or (b) that the minimum thrust of 520 ton is balanced by lateral reactions at the level of the walls above the arches and piers of the bays surrounding the tiburio, in the choir, transepts and main nave respectively. These forces have been assumed to act between 38,3 and 33,8m. Around 30m the lateral thrust from the gothic arch comes into the system. At 24m the lateral thrust from the longitudinal arches (in the transept, choir and main nave). The resultant force at the base is close to the centre of the cross-section (eccentricity 20cm) and nearly vertical (angle 1°). This calculation is carried out without considering the presence of the ties in the pointed arches, reducing the lateral thrusts.

This analysis leads to two major results, that are related to the analysis of the foundations soil in the following. In first place, if the thrust of the arches supporting the tiburio is assumed to be balanced by the walls above the transepts, nave and choir arches, the reaction at the base of the main piers of the tiburio act very close to the centre of the foundation, and with a very small angle with respect to the vertical. In second place, as a consequence, the horizontal thrusts must have been balanced by horizontal reactions of the piers surrounding the tiburio, that hence had more eccentric and inclined reactions. The analysis of this complex three-dimensional system of arches and supports has not been carried out in this work. Hence in the following the reactions at the base of the four main piers will be considered vertical and acting in the centre of the foundation. The same assumption has been made for all other piers, not considering eccentricity and inclination.

Fig.6b shows the cross section of the piers, with an outer Candoglia marble ring and an inner core of Serizzo blocks, brick and rubble. Serizzo and Candoglia have similar strength (around 100 MPa); the builders were surely relying on materials with high quality and mechanical properties. Candoglia is stiffer, and the inner core is prone to settle leaving the outer core to bear the load. Moreover the Candoglia ring is irregular in thickness. Stress concentrations can occur both because of the irregular geometry and the reduced cross-section. Although the internal forces are higher at the base, the force may be more eccentric higher up along the member, with higher local stresses. These features had profound consequences in the centuries that followed.

The construction proceeded to the front façade in 1630. The main spire was constructed in the late 18th century with an increase of load of approximately 780 tons, followed by four towers (the "*gugliotti*") at the corners of the base of the tiburio, in correspondence of the four main piers in plan. Each *gugliotto* weighs approximately 100 tons, around 3% of the total load on the main piers. Their construction though was sufficient to start serious cracking of these members, that led to the closing of the Cathedral at the end of the 19th century and repairs by substitution of the damaged blocks. These events showed that the main piers were close to failure. The repair technique used involved iron ties to bind the new parts to the old, that may have caused further damage of the sound material (Ferrari da Passano and Brivio, 1967).

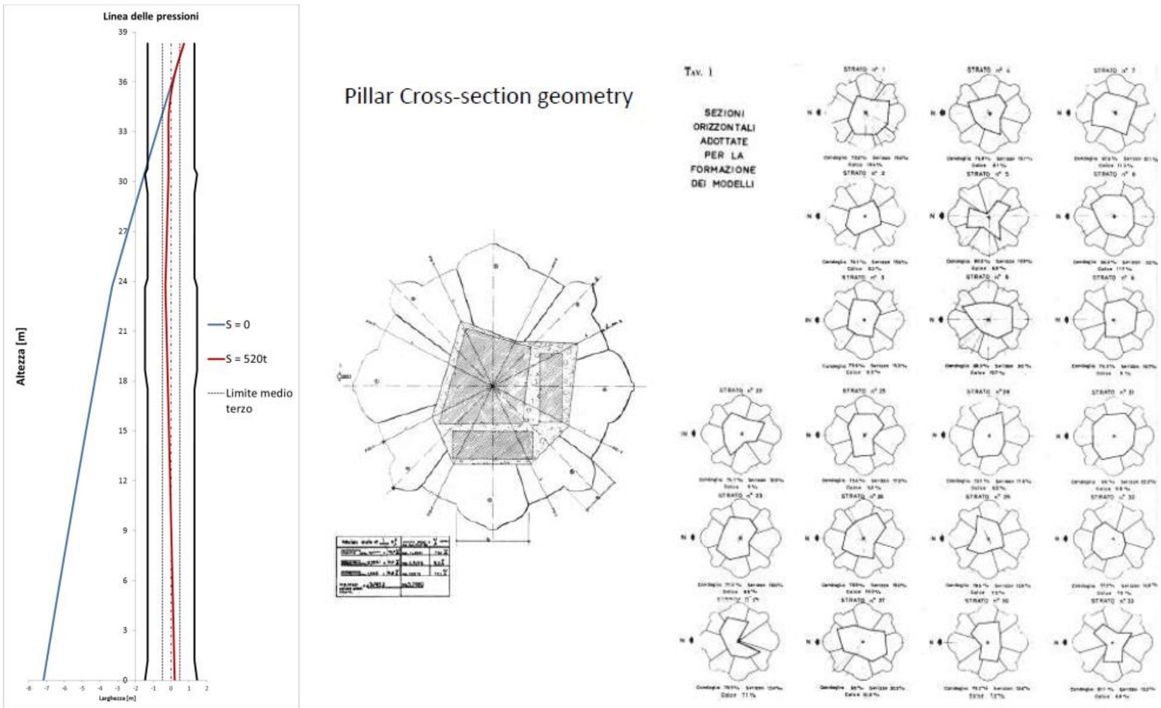


Fig.6 (a)Thrust line in the piers without (blue line) and with buttressing (red line) from the structures around the Tiburio; (b) geometry of the piers cross section (from Ferrari da Passano, 1988).

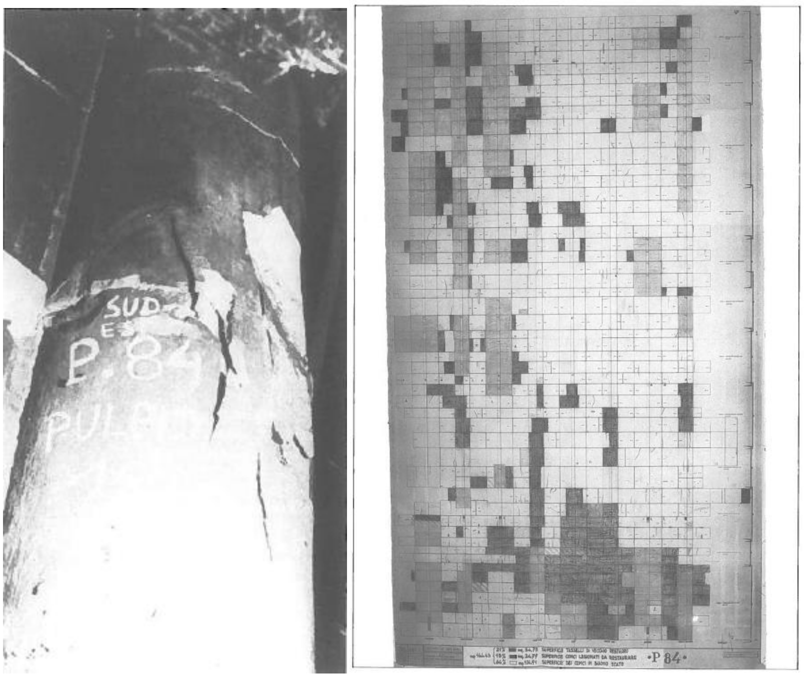


Fig.7 Damage in the piers of the tiburio: photogrpah (left); drawing of the damage pattern (dark grey. Damaged blocks; light grey: blocks damaged in the 19th century restoration; from Ferrari da Passano, 1988).

During the 20th century the use of water from many wells inside the city and the industry around Milan caused a lowering of the water table of more than 25m. This triggered subsidence in the soil with differential settlements in the Cathedral foundations and redistribution of the internal forces (Ferrari da Passano and Oberti, 1983). The piers showed extremely severe signs of distress with vertical cracks (Fig.7) and also some portions being expelled under the compression.

The remedial actions were undertaken by the Veneranda Fabbrica del Duomo with Carlo Ferrari da Passano, a team from the Politecnico di Milano lead by prof. Piero Locatelli and ISMES directed by Giulio Oberti . The main piers were encased in reinforced concrete and other 16 piers with steel cages. Diagnostics and monitoring were started. The soil subsidence was measured. Analysis showed that the foundation capacity was not overcome. The use of water from the wells was stopped, and the water table stopped lowering. Repair solutions were studied, with the final proposal made by Ferrari da Passano. The geometry of the outer rings was measured drilling holes at measurement points. New Candoglia blocks were inserted replacing the damaged ones, of larger radial dimensions where the rings were too thin – removing the interior brick and rubble before.

The safety level achieved was studied by model testing. Reduced scale models (scale factor 4,7) of the piers were contructed and tested at ISMES. A model of the whole Tiburio (scale factor 15) and supporting piers with the surrounding vaults was built, to determine the internal forces caused by the settlements.

The restoration and strengthening was concluded, and no damage has been observed ever since; regular monitoring of the deformations of the structure has started.

GEOTECHNICAL MODELLING AND SETTLEMENT ANALYSIS

The previous analysis of the history of the structure of the Tiburio provides indications on the reactions transferred on the foundations, on the basis of equilibrium calculations on the structure and assuming the minimum thrust state for the arches supporting the tiburio. In these conditions the reactions are practically vertical and close to the centroid of the column base (Fig.6). Hence the the foundation of the main piers will be assumed loaded by uniform pressures, calculated distributing the pier reactions on its top surface. The same assumption has been adopted as a simplification for all the foundations.

In this section first the geotechnical model employed for 3D settlement analyses is briefly described. For this purpose, available geological/geotechnical data have been preliminary collected from the historical archive of the DIIAR Department, Politecnico di Milano. The analysis of the soil cores extracted from seventeen boreholes (Fig. 8) allowed to define how the different soil types are spatially distributed in the area around the Cathedral.

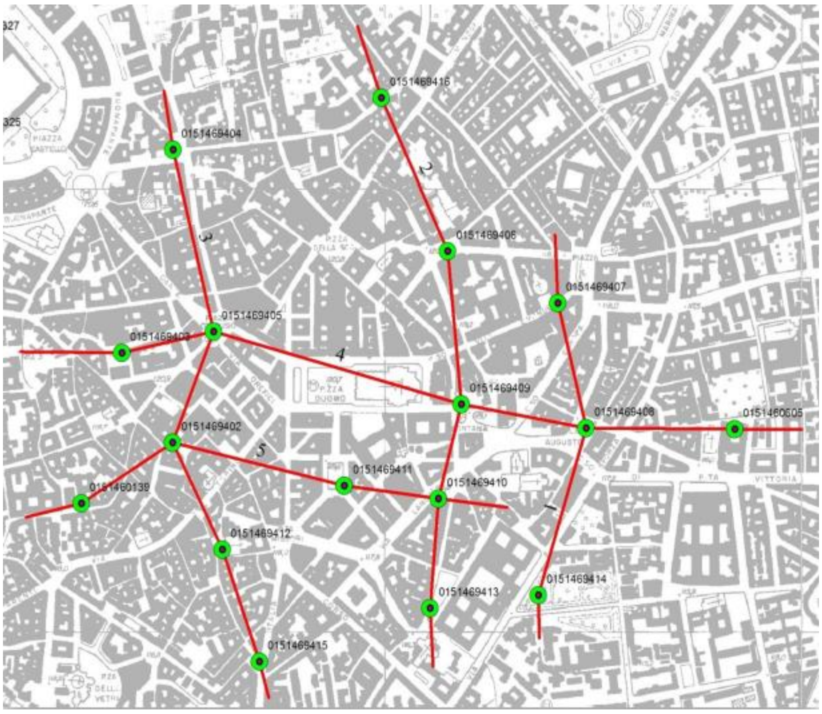


Fig.8 Plan of the boreholes used to build the model (Cathedral square at the centre).

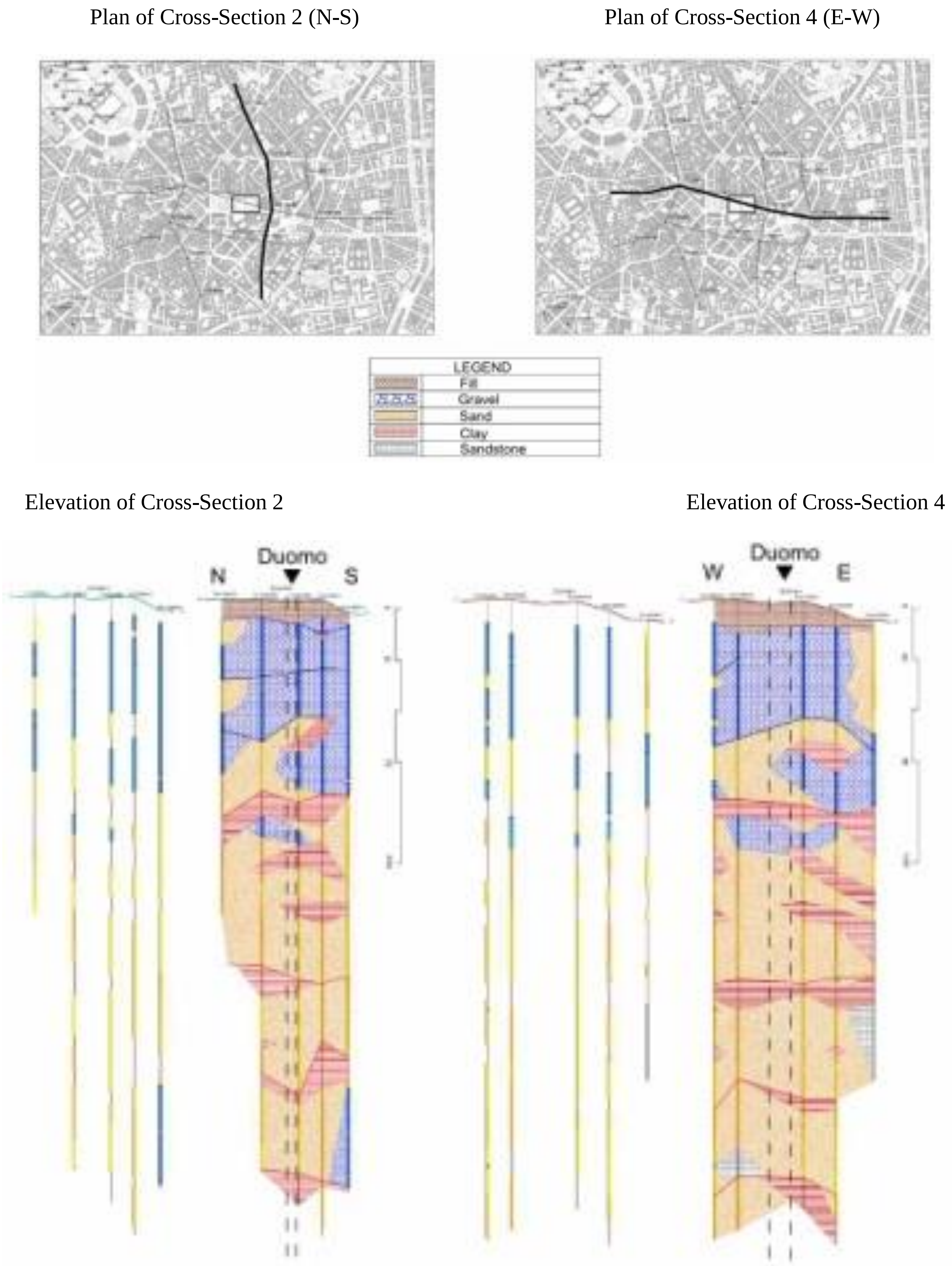


Fig. 9 – Soil profiles along two sections (see Fig.8), Section 2 (N-S) and Section 4 (E-W) down to 100m depth.

For global settlement analyses, a 600x600x120 m-sized soil domain has been discretized into tetrahedral finite elements by means of the commercial code MIDAS GTS (Fig. 10a); in Fig. 10b a detail of the soil surface area loaded by the foundations of the Cathedral columns is reported.

As is exemplified in Fig. 9, by correlating the information from different boreholes, it was possible to obtain vertical cross-sections along different horizontal orientations. The soil deposits in Milan are mostly formed by gravelly and sandy materials, including even thick clayey lenses at different locations (the legend in Fig. 9 clarifies their depth distribution over more than 100 m from the top surface).

Since no quantitative data about the mechanical soil properties right beneath the Cathedral are available, information from other studies concerning neighbouring areas have been retrieved (Niccolai, 1967). In particular, based on previous experience on granular soils in Milan, the Young modulus of both the gravelly and sandy fractions has been assumed to linearly increase along the depth, with a top surface value and a depth gradient equal to 22.5 MPa and 5.35 MPa/m, respectively. In the lack of an *ad hoc* mechanical characterization, several assumptions about the clay stiffness have been considered in this preliminary study: the results presented here have been obtained using two assumptions: homogeneous material, with the same stiffness of gravel and sand, and non-homogeneous by setting the clay Young modulus as one-tenth of the average sand/gravel stiffness.

An essential ingredient in settlement analyses is represented by the presence of the underground water, since soil settlements can be induced by not only mechanical perturbations (external loads) but also variations in the phreatic level. Indeed, as the mechanical behaviour of soils is governed by the effective stresses, it is apparent that water pressure variations can be responsible for hydraulically-induced soil deformations. This latter aspect explains the occurrence of subsidence phenomena, i.e. the soil settlements taking place when industrial/civil exploitations cause a decrease in the water table level. In Figure 11 the time variation of the phreatic level right under the Cathedral square is illustrated over the period at study (1900-1980).

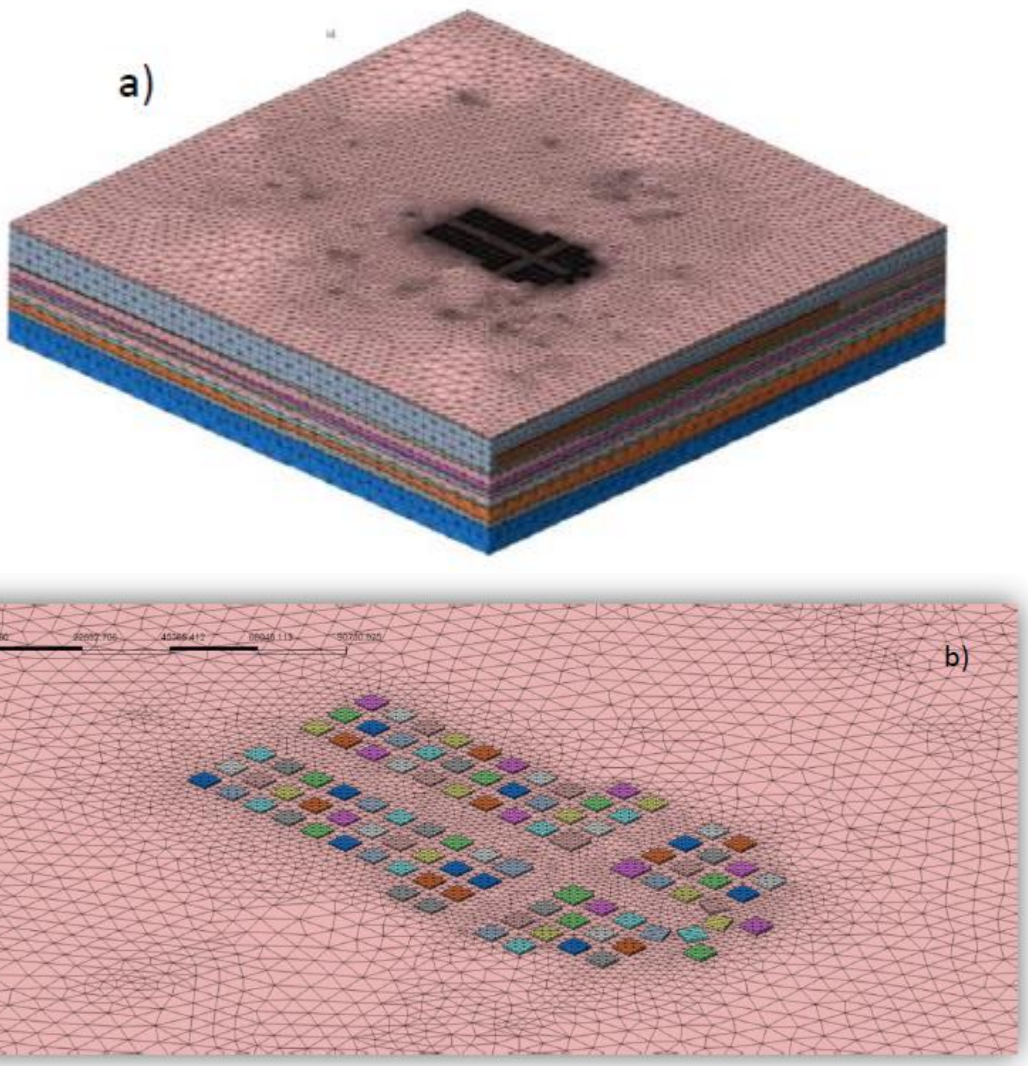


Fig. 10. (a) Discretized soil domain (600x600x120m); (b) detail of the soil surface area loaded by the foundations of the Cathedral piers.

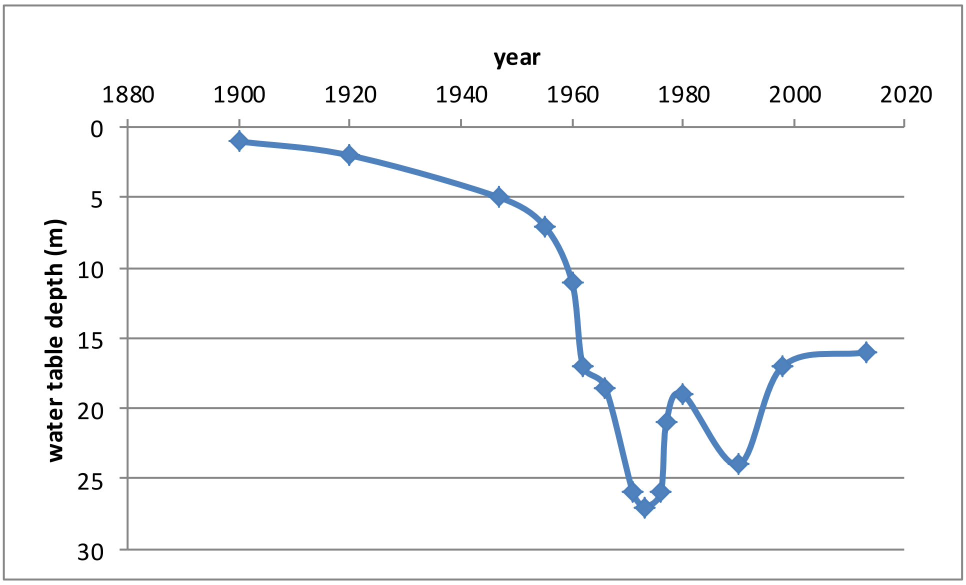
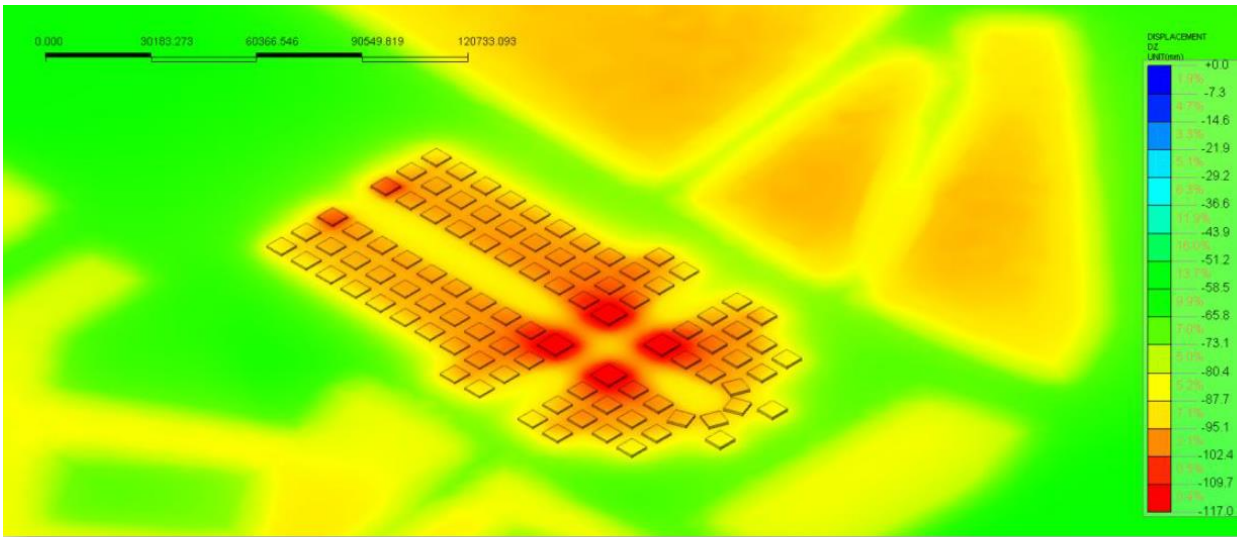
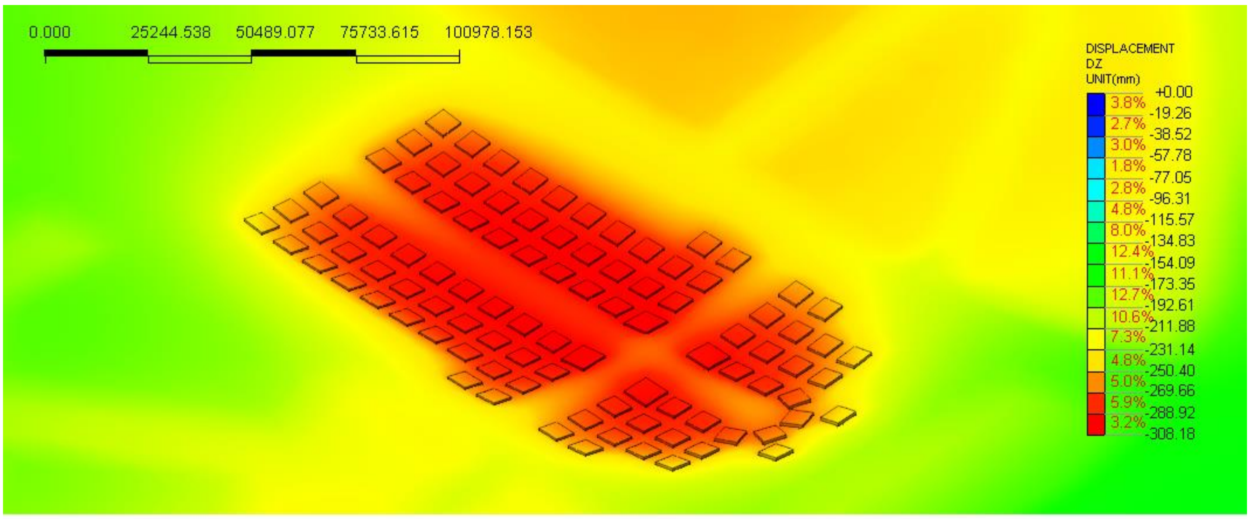


Fig. 11: Time variation of the phreatic level under the Cathedral square over the 20th century (Croce, 1985).



(a)



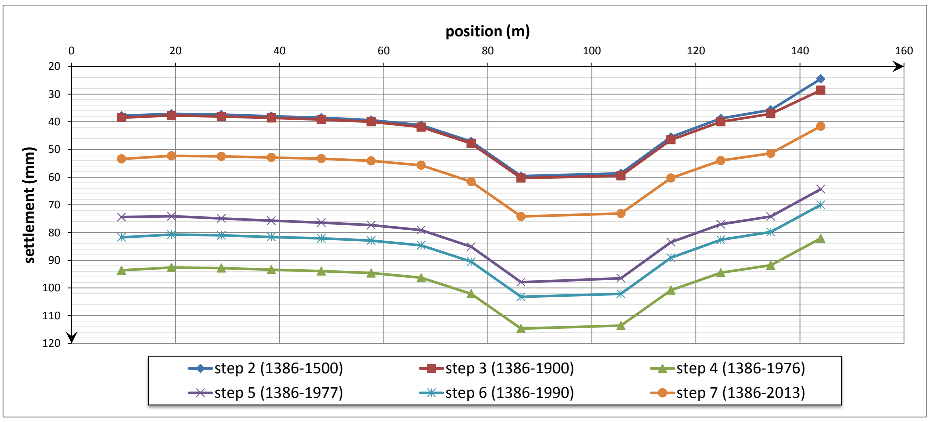
(b)

Fig.12: Contour plot of the vertical displacement at the maximum lowering of the water table: (a) homogeneous model (b) non-homogeneous.

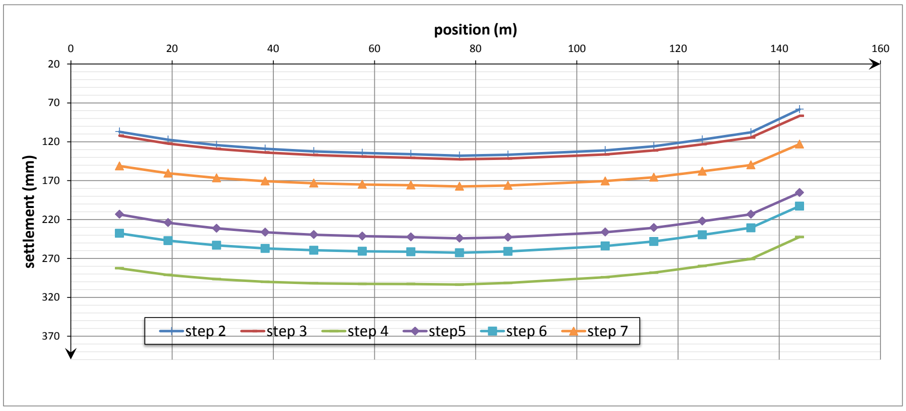
Based on these premises, the finite element settlement analysis has been carried out as a sequence of different "loading stages", reproducing in a simplified fashion the hydro-mechanical history of the soil deposit under the Cathedral. Namely:

- Step 1: gravity loading, i.e. initialization of the self weight-induced soil stresses (the resulting soil displacements are then set to nil);
- Step 2: applications of the Cathedral loads on the column foundations and perimeter walls;
- Step 3: application of the loads from the surrounding buildings, all considered as uniformly distributed surface loads;
- Steps 4-7: variations of the phreatic level.

As far as the fluctuations of the water table level are concerned, starting from an initial depth equal to 3,5 m, the values 27 (1976), 21 (1977), 24(1990), 16 m (2013) have been then consecutively applied.



(a)



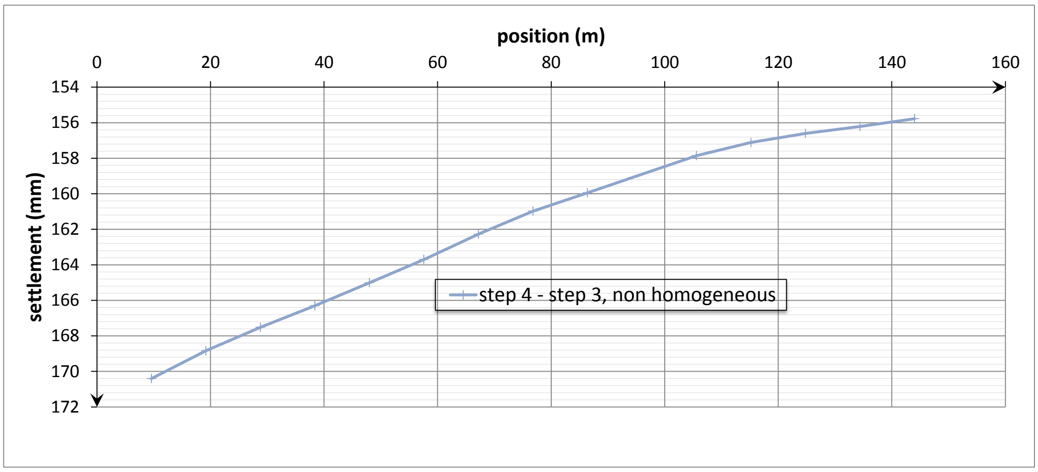
(b)

Fig.13 Longitudinal distribution of settlements for each loading step with Homogeneous (a) and non-Homogeneous (b) material assumption.

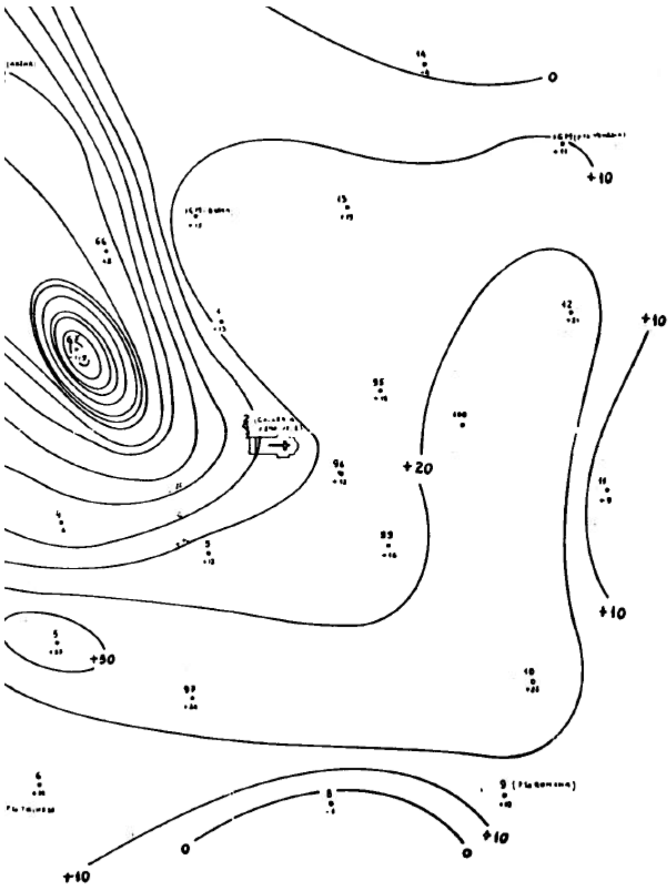
Fig. 12 provides a visual insight into the soil settlement distribution under the Cathedral and the surrounding buildings at the loading step 4. Expectedly, the most severe displacements take place under the Tiburio. The loading effects induced by the other buildings are also apparent.

Furthermore, Fig. 13a illustrates the time evolution of the soil settlements whole longitudinal profile, for a E-W line on the Northern side of the main nave, with the homogeneous material assumption. The maximum in the homogeneous case is located under the Western side of the Tiburio, with the displacements descending all the way from the Apse and Choir on one side and the Nave on the other. Hence a small differential displacement is highlighted between the Eastern and Western couples of piers of the Tiburio.

Considering the non-homogeneous soil, with reduced stiffness for the clay layers, the settlements increase largely, and the profile of the diagram changes much more gradually (Fig.13b)



(a)

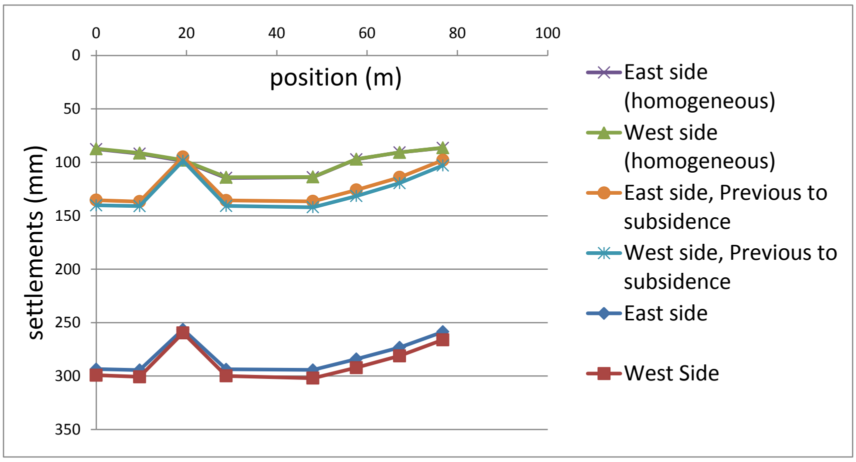


(b)

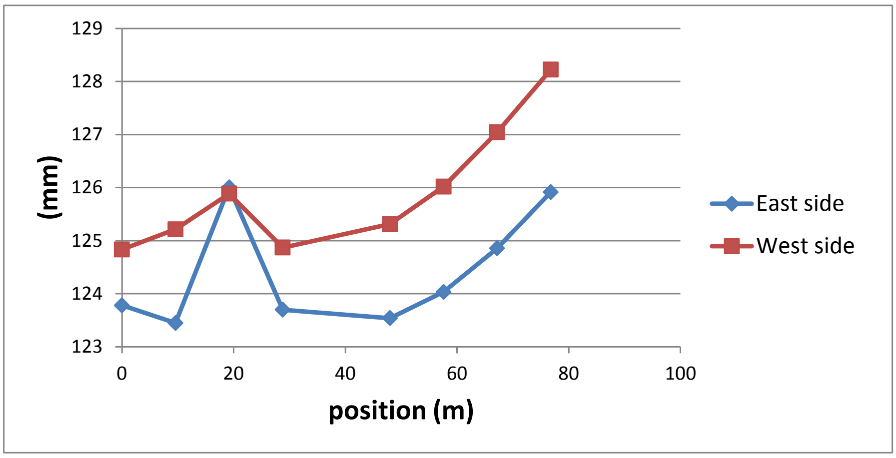
Fig.14 – Maximum settlement variation due to subsidence: (a) non homogeneous model (step4 – step3); (b) monitoring of settlements in Milan around the Duomo (in the centre), years 1950-1970 (distance beween lines of constant settlement = 10mm) (Ferrari da Passano, 1988).

Obviously, the simplistic assumption of elastic soil behaviour renders the settlement reduction upon the “hydraulic unloading” quantitatively excessive. However, while the adoption of more appropriate soil models (elasto-plastic) is needed for a more accurate settlement analysis, the results reported in this section provide a good qualitative picture of the overall settlement evolution and their spatial distribution.

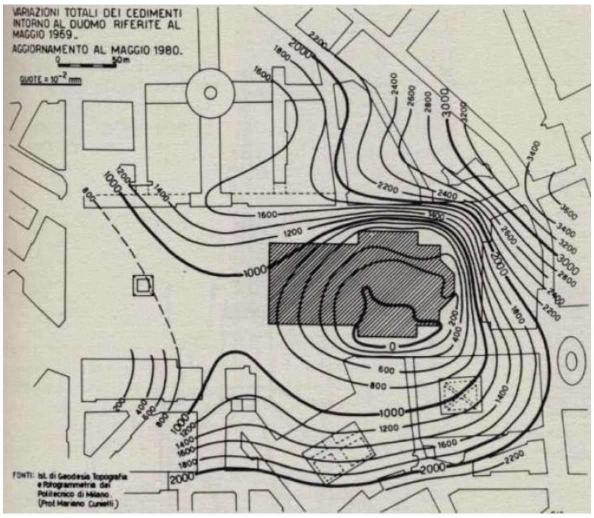
Fig.14 shows the settlements variation due to subsidence between steps 3 and 4, compared to monitoring of the same quantities on a shorter period 1950-1970, including though the fastest descent of the water table (see Fig.11). The increase from East west, with a 10mm difference along the length of the Duomo is shown both in the model and the monitoring.



(a)



(b)



(c)

Fig. 15 Non-homogeneous material assumption (reduced clay stiffness) (a) Total settlements, before and after subsidence (b) numerical variations of settlements due to subsidence with water table rising between 1967 and 1980 (c) variations of settlements monitored between 1967 and 1980, equal displacement lines 2mm (Ferrari da Passano, 1980).

More results relative to non-homogeneous soil properties assumptions are shown in Fig. 15. The displacements are those of the two parallel N-S lines of points, including the piers of the tiburio and those of the two transepts, for a total of eight piers.

The non-homogeneous material assumption with reduced clay stiffness shows (Fig.15a) a trend of settlements descending from North to South in the Northern transept. This is shown both before and after the maximum subsidence in 1967. The numerical values are total settlements, with respect to the unloaded configuration.

Fig.15b shows the variation of the settlements caused by the rising of the water table along the two lines, East and West sides of the Tiburio. Fig. 15c shows monitoring results in the period 1967-1980. A correspondence of the analyses to the experimental measurements is shown, with a descent towards the southern transept.

The results can be interpreted as follows. The soil profile model has a deep layer of clay, approximately 40m below the ground level, with thickness increasing moving from North to South. This can be considered the cause of the total displacement trend, both in the model and the monitoring.

Modelling of a single foundation

Further analyses and comparisons to the monitoring have been carried out to understand the effect of more local discontinuities of the soil profile closer to the foundation level.

Nonlinear analyses have been carried out with the commercial finite element code Tochnog (FEAT, 2014), in order to interpret more in depth the settlements under the spread foundation caused by the subsidence of the phreatic level; thus the soil-structure interaction and the elasto-plastic soil behaviour are considered. A 3D axisymmetric finite element model of one footing has been setup with the geometry of a truncated cone; the top and bottom cross-sections area are equal to those of the real member. In order to define the depth of the domain analysed the soil strength profiles in correspondence of bore holes made along the Northern side of the Duomo (Niccolai, 1967) have been used.

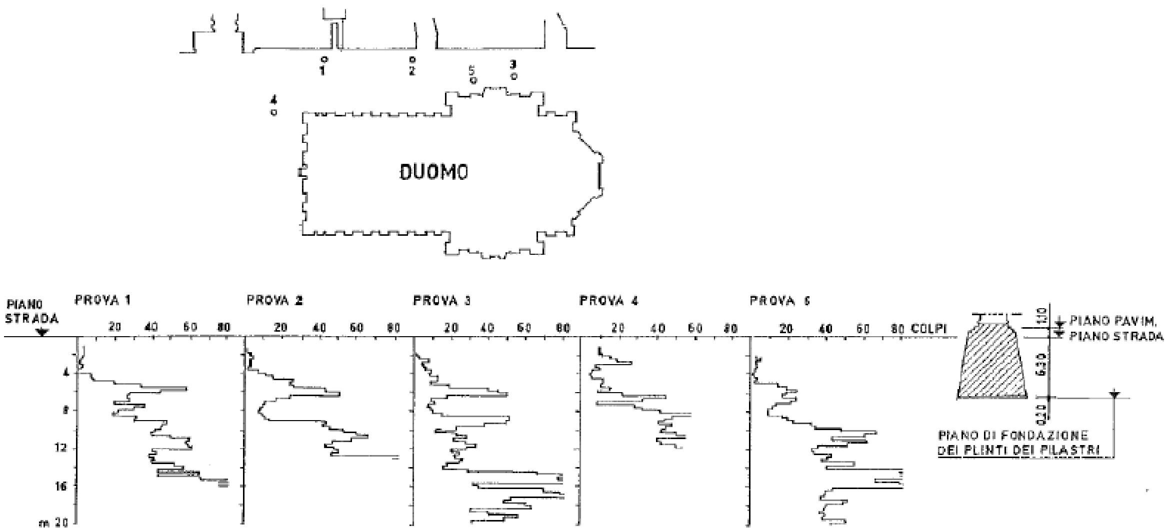


Fig.16 Soil properties – SPT results (Niccolai, 1967).

Studying these profiles, numbers 3 and 5 are particularly interesting being closer to the transept and tiburio; at a depth 16 m there is a layer with higher strength that is chosen as the lower boundary of the calculation domain. In addition, within the first 10m from the top there is a strength reduction – particularly evident in profiles 2 and 3 – showing the presence of a layer of different material. Two different soil profiles have been chosen, based on the SPT data: loose sand and the same sand with a 2m thick silt layer at the average depth of 8m. These choices simplify greatly the real profiles, but provide a simple analysis of the effects of a layer of lower strength and greater deformability.
The geometry of the foundation is shown in fig.16. The mesh is made of quadratic quadrangular elements. The masonry is modeled with a linear elastic material (E=50.000 MPa, ν=0,23), non-permeable. The interface with the soil is modeled with interface elements with a friction angle 75% of that of the soil. The soil is modeled with an elasto-plastic model, with linear elasticity (sand: E=1.e5 kPa, ν=0,23; silt: E = 8.5.e3 kPa, ν= 0.23), and non-associated Mohr-Coulomb plastic behavior (sand: φ = 32° , c = 0.1 kPa, ψ = 3°; silt: φ = 30° , c = 10 kPa, ψ = 3°).

The sand permeability is 1.e-3 m/s, the silt permeability os 1.e-8 m/s. The dry unit weight of both soils is 1,6 t/m^3 while the saturated weight is 1.8 t/m^3.

The water table is initially at a depth of 3m. After creating the initial stress state due to the self-weight of the soil and the foundation, the foundation has been loaded vertically up to the stress level transferred from the pier (Coronelli et al., 2014). This stress remains constant while the water table is lowered by 13m down to the lower border of the calculation domain in a time of 10 years, taking into consideration the consolidation process.

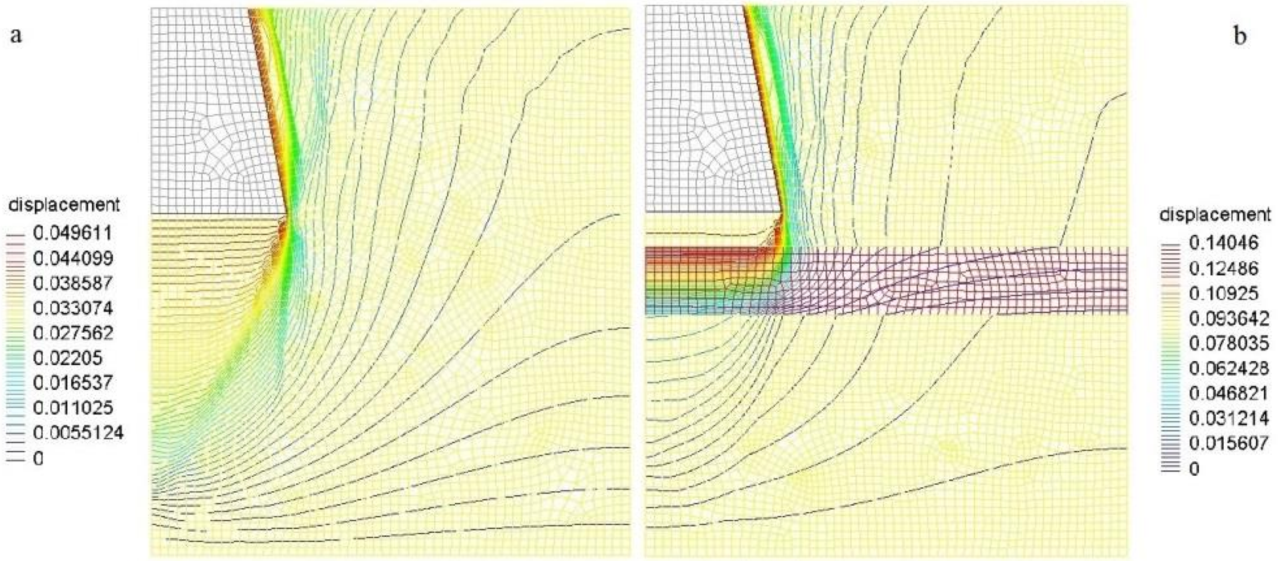


Fig. 17 Displacement values [m]: soil (a) homogeneous sand). (b) sand with silt layer.

Fig.17 shows the displacement value caused by the vertical load and the phreatic subsidence in the case of (a) sand and (b) sand with silt layer. In the homogeneous sand soil the displacement show the typical conical distribution under the foundation, in the presence of the silt layer the deformations are more concentrated in the soft layer, under the foundation during the construction stage and with a uniform distribution in the silt layer during the lowering of the water table. The displacement occurring during the construction stage is 90% of the final displacement; for this reason the displacement field is similar to that corresponding to a load on a finite portion of the soil.

In order to study the displacement during the lowering of the water Table, Fig.18 shows the displacements at the centre of the foundation during this phase for both models (a) and (b). The phreatic subsidence causes definitely higher displacements in the presence of the silt layer. The conclusion is that the soil profile close to each foundation influences significantly the foundation settlements caused by the subsidence.

Fig.19 shows the differential settlements of the piers of the tiburio, at 19.2 m distance, measured during the monitoring in the period of the phreatic subsidence under the Duomo (1966-1970). The piers on the North side (p.84 and p.85) have ascended with reference to a reference pier (p.39, the member between the two aisles in the row of piers on the West side of the Southern transept) while the two piers of the Tiburio to the South (p.74 and 75) have descended. Such difference of behavior highlights a differential settlement between these couples of members.

The global model in the previous section shows a difference between the North and South transept (Fig.15), but the markedly different behaviour of the piers of the moving from North to South is not fully reproduced. Based on the nonlinear analyses shown in this section, such difference between the North and South side of the Tiburio could be enhanced by local non homogeneity of the soil, not taken into account in the global model of the cathedral foundations in this study, together with redistribution of internal forces in the structure interacting with the soil.

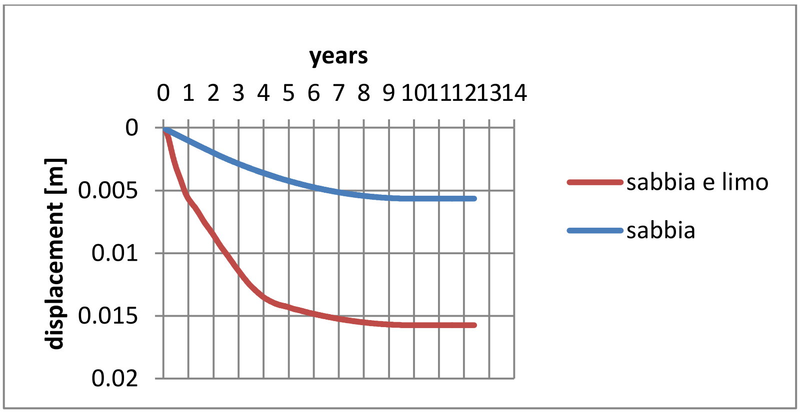


Fig. 18 – Subsidence displacements, numerical results; loose sand (blue), loose sand and silt (red).

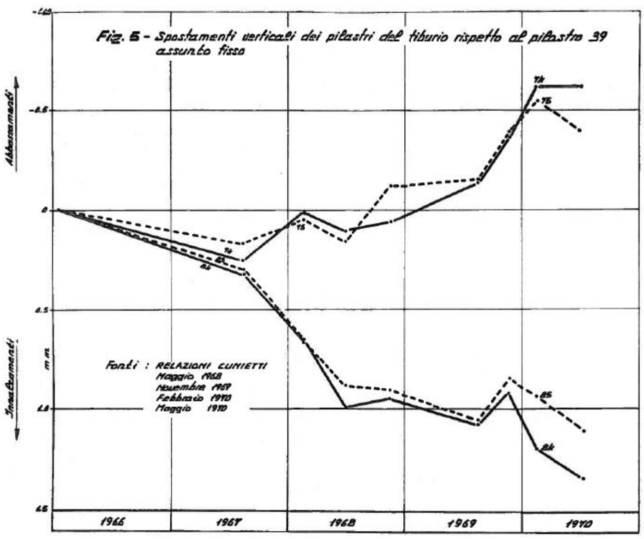


Fig.19 – Monitoring data, settlements of the piers supporting the tiburio: positive (downwards) and negative (upwards) years 1966-1970

CONCLUSIONS

The structural and construction history of the tiburio of the Duomo di Milano has been outlined. Limit analysis calculations have provided indications on the reactions on the foundation soil through the different phases of the construction up to the final configuration at the end of the 19th century.

The subsidence soil settlements of the 20th century have been analysed using a global elastic model of the whole foundation soil under the Cathedral and a more local nonlinear model of the foundation soil under one pier.

The former model used measurements on the soil profiles around the Cathedral to build a 3D finite element model including different soil layers with variations of the mechanical properties and geometry. The model of the single foundation underlines the effect of local non-homogeneity of the soil beneath.

Settlements measurement from the monitoring of the Cathedral have been compared to the models. This comparison shows that the heterogeneous soil profile with the subsidence are the causes of globally descending settlement trends from East to West along the longitudinal axis and from North to South across the transepts and Tiburio.

The models provide a basis for more refined nonlinear analyses, and study of the effects on the Cathedral of the movements of the water table in Milan.

ACKNOWLEDGEMENTS

The Authors are grateful to the Veneranda Fabbrica del Duomo and the Architect of the Fabbrica Ing. Benigno Mörlin Visconti Castiglione for the guidance in the Archives and fruitful discussions.

REFERENCES

Coronelli, D., Caggioni, B., Zanella, F. (2014)"The Cathedral of Milan: the structural history of the load bearing system*." International Journal of Architectural Heritage*, accepted for publication.

Croce A. (1985) – Antichi monumenti e città. Ricerca e conservazione. Geotechnical Engineering in Italy. An overview, 1985. Published in occasion of the ISSMFE Golden Jubilee, AGI

FEAT, 2014, "User Manual Tochnog Professional", www.feat.nl

Ferrari da Passano C., Brivio E. 1967. Contributo allo studio del Tiburio del Duomo di Milano, alcune considerazioni maturate a seguito dei lavori di restauro", Arte Lombarda, I semestre: 5-29.

Ferrari da Passano, C. (1980) – Interventi di consolidamento delle strutture del Duomo di Milano a seguito di fenomeni di subsidenza. Atti del XIV° Convegno Nazionale di Geotecnica, vol. 1,

Ferrari da Passano, C. 1988. Il Duomo rinato. Diakronia, Vigevano, 136 pp.

Ferrari da Passano, C., Oberti, G. (1983) Reconstruction des Structures du Dôme de Milan. IABSE Symposium on Strengthening of Building Structures – Diagnosis and Therapy, Venezia, 277-286.

Heyman, J. 1995. The stone skeleton, structural engineering of masonry architecture", Cambridge University Press: 159 pp.

Huerta S (2001) Mechanics of masonry vaults: the equilibrium approach. In: Historical Constructions. University of Minho, Guimaraes, pp 47–69.

Niccolai, C., 1967. Il sottosuolo del territorio di Milano, Rivista Italiana di Geotecnica, n.1: 7-33. pp. 177-186, Firenze

Vasic, M., Coronelli, D. (2014) "Limit analysis of the Duomo di Milano vaulting system" SAHC 2014, 9th Int. Conf. on Structural Anlysis of Historical Constructions, F. Pena & M. Chavez (Eds.) Mexico city, Mexico, 14-17 October 2014.